Probability & Statistics Hustle Solutions

1) $s = \sqrt{\dfrac{(5-5)^2+(4-5)^2+(4-5)^2+(6-5)^2+(6-5)^2}{5-1}} = \sqrt{\dfrac{4}{4}} = 1$

2) systematic random sample

3) $\dfrac{8!}{3!} = 8 \times 7 \times 6 \times 5 \times 4 = 6720$

4) P(Type I error) $= \alpha = 0.12$

5) $\binom{4}{2}(.6)^2(.4)^2 = 6 \times \dfrac{9}{25} x \dfrac{4}{25} = \dfrac{216}{625}$

6) mutually exclusive. Sum of vowels = 30, sum of consonants = 12. Mean $= \dfrac{42}{10} = \dfrac{21}{5} = 4.2$

7) residual $= y - \hat{y} = 26 - 31.5 = -5.5$

8) High Outlier > Q3 + 1.5IQR $= 32 + 1.5(32 - 22) = 32 + 15 = 47$. The smallest integer than can be considered a high outlier is the smallest integer greater than 47, so answer is 48.

9) Factors of 100 are 1, 2, 4, 5, 10, 20, 25, 50, 100. Six of these factors are less than 21. Therefore, the answer is $\dfrac{2}{3}$

10) $Var(X) = \sum x_i^2 \, p_i - \mu^2 = (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2) \times \dfrac{1}{6} - (3.5)^2$

$= \dfrac{91}{6} - \dfrac{49}{4} = \dfrac{35}{12}$

11) Margin of error $= \dfrac{2.76-2.34}{2} = .21$

12) Correlation coefficient $= -\sqrt{.3025} = -.55$

13) 1,1,2 3, 5, 8 13, 21, 34 ….. For every three terms, the first two terms are odd. For 2018 terms of the Fibonacci sequence, $\dfrac{2018}{3} = 3(672) + 2$ so there are $2(672) + 2 = 1346$ odd terms.

14) Let set A= {Distinct 3 letter words in alphabetical order} and let set B = {Distinct 3 letter words not in alphabetical order}. Given any element of B, there is a corresponding element in A created by just unscrambling the letters. There number of elements in A = the number of elements in B. The number of elements in B is simply $\binom{26}{3} = 2600$

15) A standardized score of -0.5 is one half a standard deviation below the mean. Therefore, with a mean of 21 and standard deviation of 4, -0.5 corresponds to an ACT score of 19.

16) Using the line of random digits looking at each pair of numbers and ignoring those outside the range of 01 to 30, the third number chosen would be 10.

17) There are 3 x 4 = 12 treatments. Therefore, with 60 experimental units, there must be 5 assigned to each treatment.

18) Let R = event of referral to a specialist   L = event of lab work
We want to find
$P[R \cap L] = P[R] + P[L] - P[R \cup L] = P[R] + P[L] - 1 + P[\sim(R \cup L)]$
$= P[R] + P[L] - 1 + P[\sim R \cap \sim L] = 0.30 + 0.40 - 1 + 0.35 = 0.05$

19) P(at least two heads) $= 1 - [P(0) + P(1)] = 1 - \left[\frac{1}{32} + \frac{5}{32}\right] = \frac{26}{32} = \frac{13}{16}$

20) $(0.5)(1.2)(\text{height}) = 1$ Therefore, height $= \frac{5}{3}$

21) df $= (r - 1)(c - 1) = (3 - 1)(4 - 1) = 2(3) = 6$

22) Median is found by solving $10 = \frac{n+2+n+3}{2}$. $n = \frac{15}{2}$. The mean is equal to the sum of the four terms divided by 4 which is $\frac{4n+10}{4}$. Substituting $\frac{15}{2}$ for n, the mean $= 10$.

23) Sample size is extraneous. Type I error is 0.05. Therefore, Type II error is 0.17. The power of a test $= 1 -$ Type II error $= 0.83$

24) The prime numbers between 1 and 30 inclusive are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. The probability is equal to 1/3

25) This is simply $1/\binom{8}{3} = 1/56$